Independent Learning A level Maths

Expectations

- You are expected to cover Chapters 1 , 2, 3 , and 5 from the Edexcel AS and A level Mathematics Pure Mathematics Year 1/AS (ISBN: 9781292183398) **before** you start the A- level mathematics course and you are provided with remote learning work to enable you to do this.
- Keep evidence of this work as this can be used to demonstrate whether you are making suitable progress
- You will be assessed on these chapters within the first term of the course and the first two chapters within the **first** half term
- If you need any additional support you can contact myself (Ms Kumi) via email ekumi@chestnutgrove.org.uk

Chapter 1 – Algebraic expressions

1. **Lesson 1: Index laws. Watch video lesson on index laws. Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

2. **Lesson 2: Index laws. Watch video lesson from 18.04 on index laws. Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

3. **Lesson 3: Index laws. Watch video lesson from 10.03 on index laws. Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

4. **Lesson 4: Index laws. Watch video lesson from 11.13 on index laws. Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

5. **Lesson 5: Manipulating indices. Watch video lesson. Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

6. **Lesson 6: Manipulating indices. Watch video lesson. Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

7. **Lesson 7: Manipulating indices. Watch video lesson. Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

8. **Lesson 8: Rationalising surds. Watch video lesson. Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

9. **Lesson 9: Rationalising surds. Watch video lesson. Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

10. **Lesson 10: Expanding brackets. Watch video Complete questions 2a, e, g, j, u and x below and mark your work (answers are in the lesson 10 answers pdf)**

2 Expand and simplify if possible:

a $5(x+1)(x-4)$	**b** $7(x-2)(2x+5)$	**c** $3(x-3)(x-3)$
d $x(x-y)(x+y)$	**e** $x(2x+y)(3x+4)$	**f** $y(x-5)(x+1)$
g $y(3x-2y)(4x+2)$	**h** $y(7-x)(2x-5)$	**i** $x(2x+y)(5x-2)$
j $x(x+2)(x+3y-4)$	**k** $y(2x+y-1)(x+5)$	**l** $y(3x+2y-3)(2x+1)$
m $x(2x+3)(x+y-5)$	**n** $2x(3x-1)(4x-y-3)$	**o** $3x(x-2y)(2x+3y+5)$
p $(x+3)(x+2)(x+1)$	**q** $(x+2)(x-4)(x+3)$	**r** $(x+3)(x-1)(x-5)$
s $(x-5)(x-4)(x-3)$	**t** $(2x+1)(x-2)(x+1)$	**u** $(2x+3)(3x-1)(x+2)$
v $(3x-2)(2x+1)(3x-2)$	**w** $(x+y)(x-y)(x-1)$	**x** $(2x-3y)^3$

11. **Lesson 11: Factorisation. Watch video Complete questions 2o – x and 3e-i, below and mark your work (answers are in the lesson 11 answers pdf)**

2 Factorise:

a x^2+4x	**b** $2x^2+6x$	**c** $x^2+11x+24$
d $x^2+8x+12$	**e** $x^2+3x-40$	**f** $x^2-8x+12$
g x^2+5x+6	**h** $x^2-2x-24$	**i** $x^2-3x-10$
j x^2+x-20	**k** $2x^2+5x+2$	**l** $3x^2+10x-8$
m $5x^2-16x+3$	**n** $6x^2-8x-8$	
o $2x^2+7x-15$	**p** $2x^4+14x^2+24$	
q x^2-4	**r** x^2-49	
s $4x^2-25$	**t** $9x^2-25y^2$	**u** $36x^2-4$
v $2x^2-50$	**w** $6x^2-10x+4$	**x** $15x^2+42x-9$

Hint For part **n**, take 2 out as a common factor first. For part **p**, let $y = x^2$.

3 Factorise completely:

a x^3+2x	**b** x^3-x^2+x	**c** x^3-5x
d x^3-9x	**e** x^3-x^2-12x	**f** x^3+11x^2+30x
g x^3-7x^2+6x	**h** x^3-64x	**i** $2x^3-5x^2-3x$
j $2x^3+13x^2+15x$	**k** x^3-4x	**l** $3x^3+27x^2+60x$

(P) **4** Factorise completely x^4-y^4. **(2 marks)**

Problem-solving
Watch out for terms that can be written as a function of a function: $x^4 = (x^2)^2$

Algebraic expressions 1B

1 a $(x + 4)(x + 7)$
$= x^2 + 7x + 4x + 28$
$= x^2 + 11x + 28$

b $(x - 3)(x + 2)$
$= x^2 + 2x - 3x - 6$
$= x^2 - x - 6$

c $(x - 2)^2$
$= (x - 2)(x - 2)$
$= x^2 - 2x - 2x + 4$
$= x^2 - 4x + 4$

d $(x - y)(2x + 3)$
$= 2x^2 + 3x - 2xy - 3y$

e $(x + 3y)(4x - y)$
$= 4x^2 - xy + 12xy - 3y^2$
$= 4x^2 + 11xy - 3y^2$

f $(2x - 4y)(3x + y)$
$= 6x^2 + 2xy - 12xy - 4y^2$
$= 6x^2 - 10xy - 4y^2$

g $(2x - 3)(x - 4)$
$= 2x^2 - 8x - 3x + 12$
$= 2x^2 - 11x + 12$

h $(3x + 2y)^2$
$= (3x + 2y)(3x + 2y)$
$= 9x^2 + 6xy + 6xy + 4y^2$
$= 9x^2 + 12xy + 4y^2$

i $(2x + 8y)(2x + 3)$
$= 4x^2 + 6x + 16xy + 24y$

j $(x + 5)(2x + 3y - 5)$
$= x(2x + 3y - 5) + 5(2x + 3y - 5)$
$= 2x^2 + 3xy - 5x + 10x + 15y - 25$
$= 2x^2 + 3xy + 5x + 15y - 25$

k $(x - 1)(3x - 4y - 5)$
$= x(3x - 4y - 5) - (3x - 4y - 5)$
$= 3x^2 - 4xy - 5x - 3x + 4y + 5$
$= 3x^2 - 4xy - 8x + 4y + 5$

l $(x - 4y)(2x + y + 5)$
$= x(2x + y + 5) - 4y(2x + y + 5)$
$= 2x^2 + xy + 5x - 8xy - 4y^2 - 20y$
$= 2x^2 + 5x - 7xy - 4y^2 - 20y$

m $(x + 2y - 1)(x + 3)$
$= x(x + 3) + 2y(x + 3) - (x + 3)$
$= x^2 + 3x + 2xy + 6y - x - 3$
$= x^2 + 2x + 2xy + 6y - 3$

n $(2x + 2y + 3)(x + 6)$
$= 2x(x + 6) + 2y(x + 6) + 3(x + 6)$
$= 2x^2 + 12x + 2xy + 12y + 3x + 18$
$= 2x^2 + 15x + 2xy + 12y + 18$

o $(4 - y)(4y - x + 3)$
$= 4(4y - x + 3) - y(4y - x + 3)$
$= 16y - 4x + 12 - 4y^2 + xy - 3y$
$= -4y^2 - 4x + 12 + xy + 13y$

p $(4y + 5)(3x - y + 2)$
$= 4y(3x - y + 2) + 5(3x - y + 2)$
$= 12xy - 4y^2 + 8y + 15x - 5y + 10$
$= 12xy - 4y^2 + 3y + 15x + 10$

q $(5y - 2x + 3)(x - 4)$
$= 5y(x - 4) - 2x(x - 4) + 3(x - 4)$
$= 5xy - 20y - 2x^2 + 8x + 3x - 12$
$= 5xy - 20y - 2x^2 + 11x - 12$

r $(4y - x - 2)(5 - y)$
$= 4y(5 - y) - x(5 - y) - 2(5 - y)$
$= 20y - 4y^2 - 5x + xy - 10 + 2y$
$= 22y - 4y^2 - 5x + xy - 10$

2 a $5(x + 1)(x - 4)$
$= (5x + 5)(x - 4)$
$= 5x^2 - 20x + 5x - 20$
$= 5x^2 - 15x - 20$

b $7(x - 2)(2x + 5)$
$= (7x - 14)(2x + 5)$
$= 14x^2 + 35x - 28x - 70$
$= 14x^2 + 7x - 70$

c $3(x - 3)(x - 3)$
$= (3x - 9)(x - 3)$
$= 3x^2 - 9x - 9x + 27$
$= 3x^2 - 18x + 27$

d $x(x - y)(x + y)$
$= (x^2 - xy)(x + y)$
$= x^3 + x^2y - x^2y - xy^2$
$= x^3 - xy^2$

2 **e** $x(2x + y)(3x + 4)$
$= (2x^2 + xy)(3x + 4)$
$= 6x^3 + 8x^2 + 3x^2y + 4xy$

f $y(x - 5)(x + 1)$
$= (xy - 5y)(x + 1)$
$= x^2y + xy - 5xy - 5y$
$= x^2y - 4xy - 5y$

g $y(3x - 2y)(4x + 2)$
$= (3xy - 2y^2)(4x + 2)$
$= 12x^2y + 6xy - 8xy^2 - 4y^2$

h $y(7 - x)(2x - 5)$
$= (7y - xy)(2x - 5)$
$= 14xy - 35y - 2x^2y + 5xy$
$= 19xy - 35y - 2x^2y$

i $x(2x + y)(5x - 2)$
$= (2x^2 + xy)(5x - 2)$
$= 10x^3 - 4x^2 + 5x^2y - 2xy$

j $x(x + 2)(x + 3y - 4)$
$= (x^2 + 2x)(x + 3y - 4)$
$= x^2(x + 3y - 4) + 2x(x + 3y - 4)$
$= x^3 + 3x^2y - 4x^2 + 2x^2 + 6xy - 8x$
$= x^3 + 3x^2y - 2x^2 + 6xy - 8x$

k $y(2x + y - 1)(x + 5)$
$= (2xy + y^2 - y)(x + 5)$
$= 2xy(x + 5) + y^2(x + 5) - y(x + 5)$
$= 2x^2y + 10xy + xy^2 + 5y^2 - xy - 5y$
$= 2x^2y + 9xy + xy^2 + 5y^2 - 5y$

l $y(3x + 2y - 3)(2x + 1)$
$= (3xy + 2y^2 - 3y)(2x + 1)$
$= 3xy(2x + 1) + 2y^2(2x + 1) - 3y\,(2x + 1)$
$= 6x^2y + 3xy + 4xy^2 + 2y^2 - 6xy - 3y$
$= 6x^2y + 4xy^2 + 2y^2 - 3xy - 3y$

m $x\,(2x + 3)(x + y - 5)$
$= (2x^2 + 3x)(x + y - 5)$
$= 2x^2(x + y - 5) + 3x(x + y - 5)$
$= 2x^3 + 2x^2y - 10x^2 + 3x^2 + 3xy - 15x$
$= 2x^3 + 2x^2y - 7x^2 + 3xy - 15x$

n $2x\,(3x - 1)(4x - y - 3)$
$= (6x^2 - 2x)(4x - y - 3)$
$= 6x^2(4x - y - 3) - 2x(4x - y - 3)$
$= 24x^3 - 6x^2y - 18x^2 - 8x^2 + 2xy + 6x$
$= 24x^3 - 6x^2y - 26x^2 + 2xy + 6x$

o $3x\,(x - 2y)(2x + 3y + 5)$
$= (3x^2 - 6xy)(2x + 3y + 5)$
$= 3x^2(2x + 3y + 5) - 6xy(2x + 3y + 5)$
$= 6x^3 + 9x^2y + 15x^2 - 12x^2y - 18xy^2 - 30xy$
$= 6x^3 + 15x^2 - 3x^2y - 18xy^2 - 30xy$

p $(x + 3)(x + 2)(x + 1)$
$= (x^2 + 2x + 3x + 6)(x + 1)$
$= (x^2 + 5x + 6)(x + 1)$
$= x^2(x + 1) + 5x(x + 1) + 6(x + 1)$
$= x^3 + x^2 + 5x^2 + 5x + 6x + 6$
$= x^3 + 6x^2 + 11x + 6$

q $(x + 2)(x - 4)(x + 3)$
$= (x^2 - 4x + 2x - 8)(x + 3)$
$= (x^2 - 2x - 8)(x + 3)$
$= x^2(x + 3) - 2x(x + 3) - 8(x + 3)$
$= x^3 + 3x^2 - 2x^2 - 6x - 8x - 24$
$= x^3 + x^2 - 14x - 24$

r $(x + 3)(x - 1)(x - 5)$
$= (x^2 - x + 3x - 3)(x - 5)$
$= (x^2 + 2x - 3)(x - 5)$
$= x^2(x - 5) + 2x(x - 5) - 3(x - 5)$
$= x^3 - 5x^2 + 2x^2 - 10x - 3x + 15$
$= x^3 - 3x^2 - 13x + 15$

s $(x - 5)(x - 4)(x - 3)$
$= (x^2 - 4x - 5x + 20)(x - 3)$
$= (x^2 - 9x + 20)(x - 3)$
$= x^2(x - 3) - 9x(x - 3) + 20(x - 3)$
$= x^3 - 3x^2 - 9x^2 + 27x + 20x - 60$
$= x^3 - 12x^2 + 47x - 60$

t $(2x + 1)(x - 2)(x + 1)$
$= (2x^2 - 4x + x - 2)(x + 1)$
$= (2x^2 - 3x - 2)(x + 1)$
$= 2x^2(x + 1) - 3x(x + 1) - 2(x + 1)$
$= 2x^3 + 2x^2 - 3x^2 - 3x - 2x - 2$
$= 2x^3 - x^2 - 5x - 2$

u $(2x + 3)(3x - 1)(x + 2)$
$= (6x^2 - 2x + 9x - 3)(x + 2)$
$= (6x^2 + 7x - 3)(x + 2)$
$= 6x^2(x + 2) + 7x(x + 2) - 3(x + 2)$
$= 6x^3 + 12x^2 + 7x^2 + 14x - 3x - 6$
$= 6x^3 + 19x^2 + 11x - 6$

2 v $(3x - 2)(2x + 1)(3x - 2)$
$= (6x^2 + 3x - 4x - 2)(3x - 2)$
$= (6x^2 - x - 2)(3x - 2)$
$= 6x^2(3x - 2) - x(3x - 2) - 2(3x - 2)$
$= 18x^3 - 12x^2 - 3x^2 + 2x - 6x + 4$
$= 18x^3 - 15x^2 - 4x + 4$

w $(x + y)(x - y)(x - 1)$
$= (x^2 - xy + xy - y^2)(x - 1)$
$= (x^2 - y^2)(x - 1)$
$= x^2(x - 1) - y^2(x - 1)$
$= x^3 - x^2 - xy^2 + y^2$

x $(2x - 3y)^3$
$= (2x - 3y)(2x - 3y)(2x - 3y)$
$= (4x^2 - 6xy - 6xy + 9y^2)(2x - 3y)$
$= (4x^2 - 12xy + 9y^2)(2x - 3y)$
$= 4x^2(2x - 3y) - 12xy(2x - 3y) + 9y^2(2x - 3y)$
$= 8x^3 - 12x^2y - 24x^2y + 36xy^2 + 18xy^2 - 27y^3$
$= 8x^3 - 36x^2y + 54xy^2 - 27y^3$

3 Shaded area
$= (x + 7)(3x - y + 4) - (x - 2)^2$
$= x(3x - y + 4) + 7(3x - y + 4) - (x - 2)(x - 2)$
$= 3x^2 - xy + 4x + 21x - 7y + 28 - x^2 + 2x + 2x - 4$
$= 2x^2 - xy + 29x - 7y + 24$

4 Volume $= (x + 2)(2x - 1)(2x + 3)$
$= (2x^2 - x + 4x - 2)(2x + 3)$
$= (2x^2 + 3x - 2)(2x + 3)$
$= 2x^2(2x + 3) + 3x(2x + 3) - 2(2x + 3)$
$= 4x^3 + 6x^2 + 6x^2 + 9x - 4x - 6$
$= 4x^3 + 12x^2 + 5x - 6 \text{ cm}^3$

5 $(2x + 5y)(3x - y)(2x + y)$
$= (6x^2 - 2xy + 15xy - 5y^2)(2x + y)$
$= (6x^2 + 13xy - 5y^2)(2x + y)$
$= 6x^2(2x + y) + 13xy(2x + y) - 5y^2(2x + y)$
$= 12x^3 + 6x^2y + 26x^2y + 13xy^2 - 10xy^2 - 5y^3$
$= 12x^3 + 32x^2y + 3xy^2 - 5y^3$
$= ax^3 + bx^2y + cxy^2 + dy^3$
Therefore, $a = 12$, $b = 32$, $c = 3$ and $d = -5$

Challenge
$(x + y)^4$
$= (x + y)(x + y)(x + y)(x + y)$
$= (x^2 + xy + xy + y^2)(x^2 + xy + xy + y^2)$
$= (x^2 + 2xy + y^2)(x^2 + 2xy + y^2)$
$= x^2(x^2 + 2xy + y^2) + 2xy(x^2 + 2xy + y^2) + y^2(x^2 + 2xy + y^2)$
$= x^4 + 2x^3y + x^2y^2 + 2x^3y + 4x^2y^2 + 2xy^3 + x^2y^2 + 2xy^3 + y^4$
$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

Algebraic expressions 1C

1 **a** $4x + 8 = 4(x + 2)$

b $6x - 24 = 6(x - 4)$

c $20x + 15 = 5(4x + 3)$

d $2x^2 + 4 = 2(x^2 + 2)$

e $4x^2 + 20 = 4(x^2 + 5)$

f $6x^2 - 18x = 6x(x - 3)$

g $x^2 - 7x = x(x - 7)$

h $2x^2 + 4x = 2x(x + 2)$

i $3x^2 - x = x(3x - 1)$

j $6x^2 - 2x = 2x(3x - 1)$

k $10y^2 - 5y = 5y(2y - 1)$

l $35x^2 - 28x = 7x(5x - 4)$

m $x^2 + 2x = x(x + 2)$

n $3y^2 + 2y = y(3y + 2)$

o $4x^2 + 12x = 4x(x + 3)$

p $5y^2 - 20y = 5y(y - 4)$

q $9xy^2 + 12x^2y = 3xy(3y + 4x)$

r $6ab - 2ab^2 = 2ab(3 - b)$

s $5x^2 - 25xy = 5x(x - 5y)$

t $12x^2y + 8xy^2 = 4xy(3x + 2y)$

u $15y - 20yz^2 = 5y(3 - 4z^2)$

v $12x^2 - 30 = 6(2x^2 - 5)$

w $xy^2 - x^2y = xy(y - x)$

x $12y^2 - 4yx = 4y(3y - x)$

2 **a** $x^2 + 4x = x(x + 4)$

b $2x^2 + 6x = 2x(x + 3)$

c $x^2 + 11x + 24 = x^2 + 8x + 3x + 24$
$= x(x + 8) + 3(x + 8)$
$= (x + 8)(x + 3)$

d $x^2 + 8x + 12 = x^2 + 2x + 6x + 12$
$= x(x + 2) + 6(x + 2)$
$= (x + 2)(x + 6)$

e $x^2 + 3x - 40 = x^2 + 8x - 5x - 40$
$= x(x + 8) - 5(x + 8)$
$= (x + 8)(x - 5)$

f $x^2 - 8x + 12 = x^2 - 2x - 6x + 12$
$= x(x - 2) - 6(x - 2)$
$= (x - 2)(x - 6)$

g $x^2 + 5x + 6 = x^2 + 3x + 2x + 6$
$= x(x + 3) + 2(x + 3)$
$= (x + 3)(x + 2)$

h $x^2 - 2x - 24 = x^2 - 6x + 4x - 24$
$= x(x - 6) + 4(x - 6)$
$= (x - 6)(x + 4)$

i $x^2 - 3x - 10 = x^2 - 5x + 2x - 10$
$= x(x - 5) + 2(x - 5)$
$= (x - 5)(x + 2)$

j $x^2 + x - 20 = x^2 - 4x + 5x - 20$
$= x(x - 4) + 5(x - 4)$
$= (x - 4)(x + 5)$

k $2x^2 + 5x + 2 = 2x^2 + x + 4x + 2$
$= x(2x + 1) + 2(2x + 1)$
$= (2x + 1)(x + 2)$

l $3x^2 + 10x - 8 = 3x^2 - 2x + 12x - 8$
$= x(3x - 2) + 4(3x - 2)$
$= (3x - 2)(x + 4)$

m $5x^2 - 16x + 3 = 5x^2 - 15x - x + 3$
$= 5x(x - 3) - (x - 3)$
$= (x - 3)(5x - 1)$

n $6x^2 - 8x - 8 = 6x^2 - 12x + 4x - 8$
$= 6x(x - 2) + 4(x - 2)$
$= (x - 2)(6x + 4)$
$= 2(x - 2)(3x + 2)$

2 o $2x^2 + 7x - 15 = 2x^2 + 10x - 3x - 15$
$= 2x(x + 5) - 3(x + 5)$
$= (x + 5)(2x - 3)$

p Put $y = x^2$
$2x^4 + 14x^2 + 24 = 2y^2 + 14y + 24$
$= 2y^2 + 6y + 8y + 24$
$= 2y(y + 3) + 8(y + 3)$
$= (y + 3)(2y + 8)$
$= (x^2 + 3)(2x^2 + 8)$
$= 2(x^2 + 3)(x^2 + 4)$

q $x^2 - 4 = x^2 - 2^2$
$= (x + 2)(x - 2)$

r $x^2 - 49 = x^2 - 7^2$
$= (x + 7)(x - 7)$

s $4x^2 - 25 = (2x)^2 - 5^2$
$= (2x + 5)(2x - 5)$

t $9x^2 - 25y^2 = (3x)^2 - (5y)^2$
$= (3x + 5y)(3x - 5y)$

u $36x^2 - 4 = 4(9x^2 - 1)$
$= 4(3x)^2 - 1^2$
$= 4(3x + 1)(3x - 1)$

v $2x^2 - 50 = 2(x^2 - 25)$
$= 2(x^2 - 5^2)$
$= 2(x + 5)(x - 5)$

w $6x^2 - 10x + 4 = 2(3x^2 - 5x + 2)$
$= 2(3x^2 - 3x - 2x + 2)$
$= 2\,(3x(x - 1) - 2(x - 1))$
$= 2(x - 1)(3x - 2)$

x $15x^2 + 42x - 9 = 3(5x^2 + 14x - 3)$
$= 3(5x^2 - x + 15x - 3)$
$= 3\,(x(5x - 1) + 3(5x - 1))$
$= 3(5x - 1)(x + 3)$

3 a $x^3 + 2x = x(x^2 + 2)$

b $x^3 - x^2 + x = x(x^2 - x + 1)$

c $x^3 - 5x = x(x^2 - 5)$

d $x^3 - 9x = x(x^2 - 9)$
$= x(x^2 - 3^2)$
$= x(x + 3)(x - 3)$

3 e $x^3 - x^2 - 12x = x(x^2 - x - 12)$
$= x(x^2 - 4x + 3x - 12)$
$= x(x(x - 4) + 3(x - 4))$
$= x(x - 4)(x + 3)$

f $x^3 + 11x^2 + 30x = x(x^2 + 11x + 30)$
$= x(x^2 + 5x + 6x + 30)$
$= x(x(x + 5) + 6(x + 5))$
$= x(x + 5)(x + 6)$

g $x^3 - 7x^2 + 6x = x(x^2 - 7x + 6)$
$= x(x^2 - x - 6x + 6)$
$= x(x\,(x - 1) - 6(x - 1))$
$= x(x - 1)(x - 6)$

h $x^3 - 64x = x(x^2 - 64)$
$= x(x^2 - 8^2)$
$= x(x + 8)(x - 8)$

i $2x^3 - 5x^2 - 3x = x(2x^2 - 5x - 3)$
$= x(2x^2 + x - 6x - 3)$
$= x(x\,(2x + 1) - 3(2x + 1))$
$= x(2x + 1)(x - 3)$

j $2x^3 + 13x^2 + 15x = x(2x^2 + 13x + 15)$
$= x(2x^2 + 3x + 10x + 15)$
$= x(x(2x + 3) + 5(2x + 3))$
$= x(2x + 3)(x + 5)$

k $x^3 - 4x = x(x^2 - 4)$
$= x(x^2 - 2^2)$
$= x(x + 2)(x - 2)$

l $3x^3 + 27x^2 + 60x = 3x(x^2 + 9x + 20)$
$= 3x(x^2 + 4x + 5x + 20)$
$= 3x(x(x + 4) + 5(x + 4))$
$= 3x(x + 4)(x + 5)$

4 $x^4 - y^4 = (x^2)^2 - (y^2)^2$
$= (x^2 + y^2)(x^2 - y^2)$
$= (x^2 + y^2)(x + y)(x - y)$

5 $6x^3 + 7x^2 - 5x = x(6x^2 + 7x - 5)$
$= x(6x^2 + 10x - 3x - 5)$
$= x(2x(3x + 5) - (3x + 5))$
$= x(3x + 5)(\,2x - 1)$

Challenge

$4x^4 - 13x^2 + 9 = (4x^4 - 4x^2 - 9x^2 + 9)$
$= 4x^2(x^2 - 1) - 9(x^2 - 1)$
$= (x^2 - 1)(4x^2 - 9)$
$= (x^2 - 1^2)(\,2x)^2 - 3^2)$
$= (x + 1)(x - 1)(2x + 3)(2x - 3)$

Independent Learning A level Maths

Expectations

- You are expected to cover Chapters 1 , 2, 3 and 5 from the Edexcel AS and A level Mathematics Pure Mathematics Year 1/AS (ISBN: 9781292183398) **before** you start the A- level mathematics course and you are provided with remote learning work to enable you to do this.
- Keep evidence of this work as this can be used to demonstrate whether you are making suitable progress
- You will be assessed on these chapters within the first term of the course and the first two chapters within the **first** half term
- If you need any additional support you can contact myself (Ms Kumi) via email ekumi@chestnutgrove.org.uk

Chapter 2 – Quadratics

1. **Lesson 1: Solving simple quadratics (a=1) by factorising. Watch video lesson. Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

2. **Lesson 2: Solving more complex quadratics by factorising. Watch video lesson . Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work. EXTRA SUPPORT: If you need to refresh your memory on factorising quadratics when a is not 1 then watch video first.**

3. **Lesson 3: Solving quadratics using the quadratic formula. Watch video lesson. Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

4. **Lesson 4: Solving quadratics by completing the square. Watch video lesson on completing the square. Copy examples, complete all the my turn and exercise questions. Make sure you mark all your work. Then watch video on solving quadratics by completing the square.**

5. **Lesson 5: Functions. Watch video lesson. Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

6. **Lesson 6: Sketching quadratic graphs. Watch video lesson. Copy the example and make notes. Extra support: Watch video on maximum and minimum points. Complete questions below . Full solutions are in the lesson 6 solutions document.**

Exercise 2F

1 Sketch the graphs of the following equations. For each graph, show the coordinates of the point(s) where the graph crosses the coordinate axes, and write down the coordinate of the turning point and the equation of the line of symmetry.

a $y = x^2 - 6x + 8$ **b** $y = x^2 + 2x - 15$ **c** $y = 25 - x^2$ **d** $y = x^2 + 3x + 2$

e $y = -x^2 + 6x + 7$ **f** $y = 2x^2 + 4x + 10$ **g** $y = 2x^2 + 7x - 15$ **h** $y = 6x^2 - 19x + 10$

E/P **3** The graph of $y = ax^2 + bx + c$ has a minimum at $(5, -3)$ and passes through $(4, 0)$. Find the values of a, b and c. **(3 marks)**

7. **Lesson 7: The discriminant. Watch video lesson. Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work. Then watch video from 4.32 on applying the discriminant. Complete questions below . Full solutions are in the lesson 7 solutions document.**

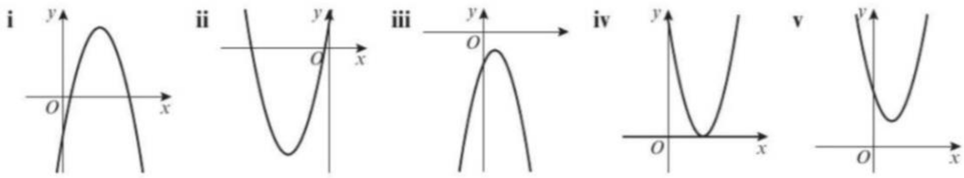
Exercise 2G

1 a Calculate the value of the discriminant for each of these five functions:

i $f(x) = x^2 + 8x + 3$ ii $g(x) = 2x^2 - 3x + 4$ iii $h(x) = -x^2 + 7x - 3$

iv $j(x) = x^2 - 8x + 16$ v $k(x) = 2x - 3x^2 - 4$

b Using your answers to part **a**, match the same five functions to these sketch graphs.



E/P 2 Find the values of k for which $x^2 + 6x + k = 0$ has two real solutions. **(2 marks)**

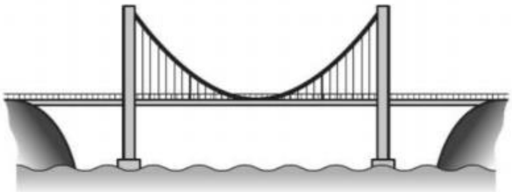
E/P 3 Find the value of t for which $2x^2 - 3x + t = 0$ has exactly one solution. **(2 marks)**

E/P 4 Given that the function $f(x) = sx^2 + 8x + s$ has equal roots, find the value of the positive constant s. **(2 marks)**

8. **Lesson 8: Modelling Quadratics . Watch video lesson. Copy the example and make notes. Then watch video for another example. Complete questions below , solutions are in the lesson 8 document.**

Exercise 2H

E/P 1 The diagram shows a section of a suspension bridge carrying a road over water.



Problem-solving

For part **a**, make sure your answer is in the context of the model.

The height of the cables above water level in metres can be modelled by the function $h(x) = 0.000\,12x^2 + 200$, where x is the displacement in metres from the centre of the bridge.

a Interpret the meaning of the constant term 200 in the model. **(1 mark)**

b Use the model to find the two values of x at which the height is 346 m. **(3 marks)**

c Given that the towers at each end are 346 m tall, use your answer to part **b** to calculate the length of the bridge to the nearest metre. **(1 mark)**

E/P 2 A car manufacturer uses a model to predict the fuel consumption, y miles per gallon (mpg), for a specific model of car travelling at a speed of x mph.

$$y = -0.01x^2 + 0.975x + 16, \; x > 0$$

a Use the model to find two speeds at which the car has a fuel consumption of 32.5 mpg. **(3 marks)**

b Rewrite y in the form $A - B(x - C)^2$, where A, B and C are constants to be found. **(3 marks)**

c Using your answer to part **b**, find the speed at which the car has the greatest fuel efficiency. **(1 mark)**

d Use the model to calculate the fuel consumption of a car travelling at 120 mph. Comment on the validity of using this model for very high speeds. **(2 marks)**

Quadratics 2F

1 a $y = x^2 - 6x + 8$

As $a = 1$ is positive, the graph has a $\cup$ shape and a minimum point.

When $x = 0$, $y = 8$, so the graph crosses the y-axis at (0, 8).

When $y = 0$,

$x^2 - 6x + 8 = 0$

$(x - 2)(x - 4) = 0$

$x = 2$ or $x = 4$, so the graph crosses the x-axis at (2, 0) and (4, 0).

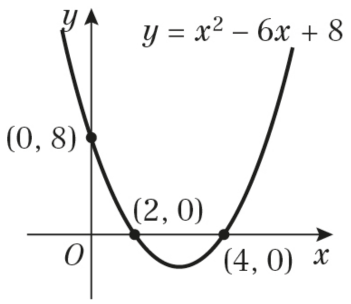
Completing the square:

$x^2 - 6x + 8 = (x - 3)^2 - 9 + 8$

$= (x - 3)^2 - 1$

So the minimum point has coordinate (3, −1).

The sketch of the graph is:



b $y = x^2 + 2x - 15$

As $a = 1$ is positive, the graph has a $\cup$ shape and a minimum point.

When $x = 0$, $y = -15$, so the graph crosses the y-axis at (0, −15).

When $y = 0$,

$x^2 + 2x - 15 = 0$

$(x - 3)(x + 5) = 0$

$x = 3$ or $x = -5$, so the graph crosses the x-axis at (3, 0) and (−5, 0).

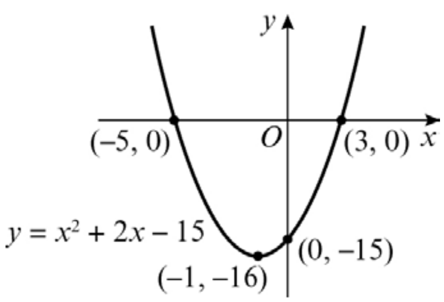
Completing the square:

$x^2 + 2x - 15 = (x + 1)^2 - 1 - 15$

$= (x + 1)^2 - 16$

So the minimum point has coordinate (−1, −16).

The sketch of the graph is:



c $y = 25 - x^2$

As $a = -1$ is negative, the graph has a $\cap$ shape and a maximum point.

When $x = 0$, $y = 25$, so the graph crosses the y-axis at (0, 25).

When $y = 0$,

$25 - x^2 = 0$

$(5 + x)(5 - x) = 0$

$x = -5$ or $x = 5$, so the graph crosses the x-axis at (−5, 0) and (5, 0).

Completing the square:

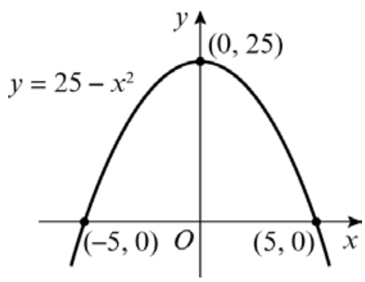
$25 - x^2 = -x^2 + 0x + 25$

$= -(x^2 - 0x - 25)$

$= -(x - 0)^2 + 25$

So the maximum point has coordinate (0, 25).

The sketch of the graph is:



d $y = x^2 + 3x + 2$

As $a = 1$ is positive, the graph has a $\cup$ shape and a minimum point.

When $x = 0$, $y = 2$, so the graph crosses the y-axis at (0, 2).

When $y = 0$,

$x^2 + 3x + 2 = 0$

$(x + 2)(x + 1) = 0$

$x = -2$ or $x = -1$, so the graph crosses the x-axis at (−2, 0) and (−1, 0).

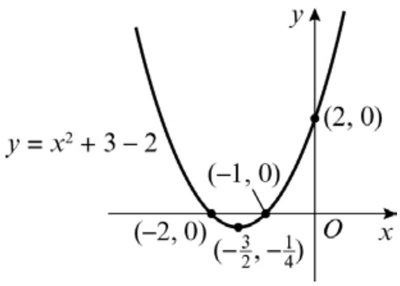
Completing the square:

$$x^2 + 3x + 2 = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + 2$$

$$= \left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$$

So the minimum point has coordinate $\left(-\frac{3}{2}, -\frac{1}{4}\right)$.

1 d The sketch of the graph is:



e $y = -x^2 + 6x + 7$

As $a = -1$ is negative, the graph has a ∩ shape and a maximum point.

When $x = 0$, $y = 7$, so the graph crosses the y-axis at (0, 7).

When $y = 0$,

$-x^2 + 6x + 7 = 0$

$(-x - 1)(x - 7) = 0$

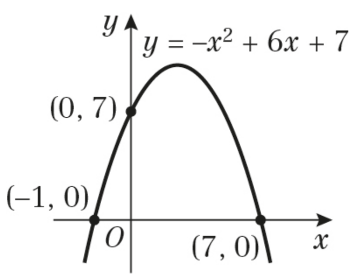
$x = -1$ or $x = 7$, so the graph crosses the x-axis at (−1, 0) and (7, 0).

Completing the square:

$$-x^2 + 6x + 7 = -(x^2 - 6x) + 7$$
$$= -((x-3)^2 - 9) + 7$$
$$= -(x-3)^2 + 16$$

So the maximum point has coordinate (3, 16).

The sketch of the graph is:



f $y = 2x^2 + 4x + 10$

As $a = 2$ is positive, the graph has a ∪ shape and a minimum point.

When $x = 0$, $y = 10$, so the graph crosses the y-axis at (0, 10).

When $y = 0$,

$2x^2 + 4x + 10 = 0$

Using the quadratic formula,

$$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(10)}}{2 \times 2}$$

$$x = \frac{-4 \pm \sqrt{-64}}{4}$$

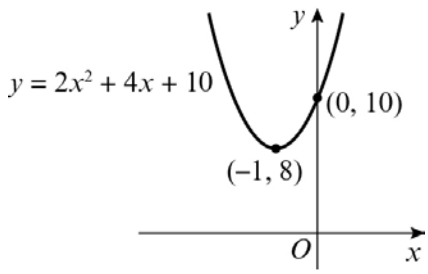
There are no real solutions, so the graph does not cross the x-axis.

f Completing the square:

$$2x^2 + 4x + 10 = 2(x^2 + 2x) + 10$$
$$= 2((x + 1)^2 - 1) + 10$$
$$= 2(x + 1)^2 + 8$$

So the minimum point has coordinate (−1, 8).

The sketch of the graph is:



g $y = 2x^2 + 7x - 15$

As $a = 2$ is positive, the graph has a ∪ shape and a minimum point.

When $x = 0$, $y = -15$, so the graph crosses the y-axis at (0, −15).

When $y = 0$,

$2x^2 + 7x - 15 = 0$

$(2x - 3)(x + 5) = 0$

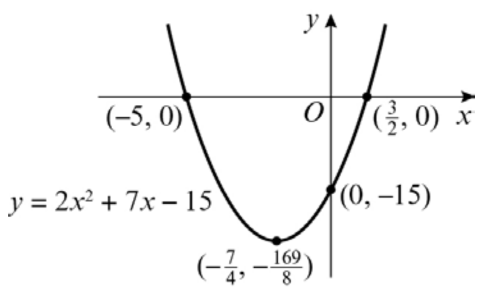
$x = \frac{3}{2}$ or $x = -5$, so the graph crosses the x-axis at $\left(\frac{3}{2}, 0\right)$ and (−5, 0).

Completing the square:

$$2x^2 + 7x - 15 = 2\left(x^2 + \frac{7}{2}x\right) - 15$$
$$= 2\left(\left(x + \frac{7}{4}\right)^2 - \frac{49}{16}\right) - 15$$
$$= 2\left(x + \frac{7}{4}\right)^2 - \frac{169}{8}$$

So the minimum point has coordinate $\left(-\frac{7}{4}, -\frac{169}{8}\right)$.

The sketch of the graph is:



1 h $y = 6x^2 - 19x + 10$

As $a = 6$ is positive, the graph has a $\vee$ shape and a minimum point.

When $x = 0$, $y = 10$, so the graph crosses the y-axis at (0, 10).

When $y = 0$,

$6x^2 - 19x + 10 = 0$

$(3x - 2)(2x - 5) = 0$

$x = \frac{2}{3}$ or $x = \frac{5}{2}$, so the graph crosses the x-axis at $\left(\frac{2}{3}, 0\right)$ and $\left(\frac{5}{2}, 0\right)$.

Completing the square:

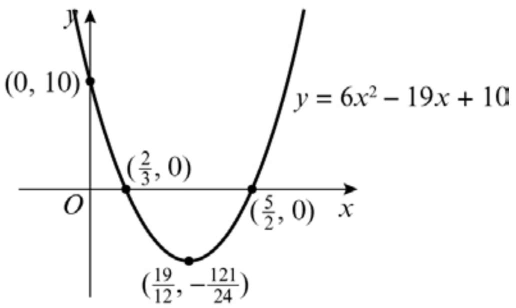
$$6x^2 - 19x + 10 = 6\left(x^2 - \frac{19}{6}x\right) + 10$$

$$= 6\left(\left(x - \frac{19}{12}\right)^2 - \frac{361}{144}\right) + 10$$

$$= 6\left(x - \frac{19}{12}\right)^2 - \frac{121}{24}$$

So the minimum point has coordinate $\left(\frac{19}{12}, -\frac{121}{24}\right)$.

The sketch of the graph is:



i $y = 4 - 7x - 2x^2$

As $a = -2$ is negative, the graph has a $\wedge$ shape and a maximum point.

When $x = 0$, $y = 4$, so the graph crosses the y-axis at (0, 4).

When $y = 0$,

$-2x^2 - 7x + 4 = 0$

$(-2x + 1)(x + 4) = 0$

$x = \frac{1}{2}$ or $x = -4$, so the graph crosses the x-axis at $\left(\frac{1}{2}, 0\right)$ and (−4, 0).

Completing the square:

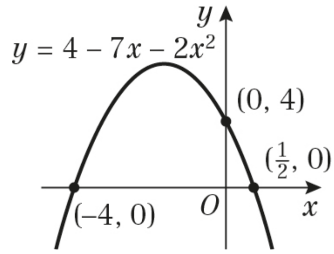
$$-2x^2 - 7x + 4 = -2\left(x^2 + \frac{7}{2}x\right) + 4$$

$$= -2\left(\left(x + \frac{7}{4}x\right)^2 - \frac{49}{16}\right) + 4$$

$$= -2\left(x + \frac{7}{4}x\right)^2 + \frac{81}{8}$$

i So the maximum point has coordinate $\left(-\frac{7}{4}, \frac{81}{8}\right)$.

The sketch of the graph is:



j $y = 0.5x^2 + 0.2x + 0.02$

As a = 0.5 is positive, the graph has a $\vee$ shape and a minimum point.

When $x = 0$, $y = 0.02$, so the graph crosses the y-axis at (0, 0.02).

When $y = 0$,

$0.5x^2 + 0.2x + 0.02 = 0$

Using the quadratic formula,

$$x = \frac{-0.2 \pm \sqrt{0.2^2 - 4(0.5)(0.02)}}{2 \times 0.5}$$

$$x = -0.2 \pm \sqrt{0}$$

$$= -0.2$$

There is only one solution, so the graph touches the x-axis.

Completing the square:

$0.5x^2 + 0.2x + 0.02$

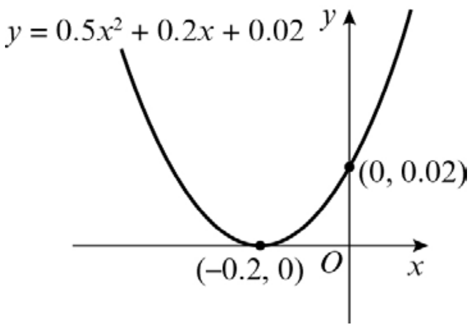
$$= 0.5(x^2 + 0.4x) + 0.02$$

$$= 0.5((x + 0.2)^2 - 0.04) + 0.02$$

$$= 0.5(x + 0.2)^2$$

So the minimum point has coordinate (−0.2, 0).

The sketch of the graph is:



2 a The graph crosses the y-axis at (0, 15), so $c = 15$.
The graph crosses the x-axis at (3, 0) and (5, 0) and has a minimum value.
$(x - 3)(x - 5) = 0$
$x^2 - 8x + 15 = 0$
$a = 1$, $b = -8$ and $c = 15$

b The graph crosses the y-axis at (0, 10), so $c = 10$.
The graph crosses the x-axis at (−2, 0) and (5, 0) and has a maximum value.
$-(x + 2)(x - 5) = 0$
$-x^2 + 3x + 10 = 0$
$a = -1$, $b = 3$ and $c = 10$

c The graph crosses the y-axis at (0, −18), so $c = -18$.
The graph crosses the x-axis at (−3, 0) and (3, 0) and has a minimum value.
$(x + 3)(x - 3) = 0$
$x^2 + 0x - 9 = 0$
But $c = -18$, not −9, so $2(x^2 + 0x - 9) = 0$
$a = 2$, $b = 0$ and $c = -18$

d The graph crosses the y-axis at (0, −1), so $c = -1$.
The graph crosses the x-axis at (−1, 0) and (4, 0) and has a minimum value.
$(x + 1)(x - 4) = 0$
$x^2 - 3x - 4 = 0$
But $c = -1$, not −4, so $\frac{1}{4}(x^2 - 3x - 4) = 0$
$a = \frac{1}{4}$, $b = -\frac{3}{4}$ and $c = -1$

3 Minimum value = (5, −3), so the line of symmetry is at $x = 5$.
The reflection of (4, 0) in the line $y = 5$ is (6, 0).
$(x - 6)(x - 4) = 0$
$x^2 - 10x + 24 = 0$
Completing the square:
$x^2 - 10x + 24 = (x - 5)^2 - 25 + 24$
$= (x - 5)^2 - 1$
But the minimum value is (5, − 3), therefore:
$y = 3(x - 5)^2 - 3$
$= 3x^2 - 30x + 72$
$a = 3$, $b = -30$ and $c = 72$

Quadratics 2G

1 a i $f(x) = x^2 + 8x + 3$
$b^2 - 4ac$
$= 8^2 - 4(1)(3)$
$= 64 - 12$
$= 52$

ii $g(x) = 2x^2 - 3x + 4$
$b^2 - 4ac$
$= (-3)^2 - 4(2)(4)$
$= 9 - 32$
$= -23$

iii $h(x) = -x^2 + 7x - 3$
$b^2 - 4ac$
$= 7^2 - 4(-1)(-3)$
$= 49 - 12$
$= 37$

iv $j(x) = x^2 - 8x + 16$
$b^2 - 4ac$
$= (-8)^2 - 4(1)(16)$
$= 64 - 64$
$= 0$

v $k(x) = 2x - 3x^2 - 4$
$= -3x^2 + 2x - 4$
$b^2 - 4ac$
$= (2)^2 - 4(-3)(-4)$
$= 4 - 48$
$= -44$

b i This graph has two distinct real roots and has a maximum, so $a < 0$: h(x).

ii This graph has two distinct real roots and has a minimum, so $a > 0$: f(x).

iii This graph has no real roots and has a maximum, so $a < 0$: k(x).

iv This graph has one repeated root and has a minimum, so $a > 0$: j(x).

v This graph has no real roots and has a minimum, so $a > 0$: g(x).

2 $x^2 + 6x + k = 0$
$a = 1$, $b = 6$ and $c = k$
For two real solutions, $b^2 - 4ac > 0$
$6^2 - 4 \times 1 \times k > 0$
$36 - 4k > 0$
$36 > 4k$
$9 > k$
So $k < 9$

3 $2x^2 - 3x + t = 0$
$a = 2$, $b = -3$ and $c = t$
For exactly one solution, $b^2 - 4ac = 0$
$(-3)^2 - 4 \times 2 \times t = 0$
$9 - 8t = 0$
So $t = \frac{9}{8}$

4 $f(x) = sx^2 + 8x + s$
$a = s$, $b = 8$ and $c = s$
For equal solutions, $b^2 - 4ac = 0$
$8^2 - 4 \times s \times s = 0$
$64 - 4s^2 = 0$
$64 = 4s^2$
$16 = s^2$
So $s = \pm 4$
The positive solution is $s = 4$.

5 $3x^2 - 4x + k = 0$
$a = 3$, $b = -4$ and $c = k$
For no real solutions, $b^2 - 4ac < 0$
$(-4)^2 - 4 \times 3 \times k < 0$
$16 - 12k < 0$
$16 < 12k$
$4 < 3k$
So $k > \frac{4}{3}$

6 a $g(x) = x^2 + 3px + (14p - 3) = 0$
$a = 1$, $b = 3p$ and $c = 14p - 3$
For two equal roots, $b^2 - 4ac = 0$
$(3p)^2 - 4 \times 1 \times (14p - 3) = 0$
$9p^2 - 56p + 12 = 0$
$(p - 6)(9p - 2) = 0$
$p = 6$ or $p = \frac{2}{9}$
p is an integer, so $p = 6$

6 b When $p = 6$,
$x^2 + 3px + (14p - 3)$
$= x^2 + 3(6)x + (14(6) - 3)$
$= x^2 + 18x + 81$
$x^2 + 18x + 81 = 0$
$(x + 9)(x + 9) = 0$
So $x = -9$

7 a $h(x) = 2x^2 + (k + 4)x + k$
$a = 2$, $b = k + 4$ and $c = k$
$b^2 - 4ac = (k + 4)^2 - 4 \times 2 \times k$
$= k^2 + 8k + 16 - 8k = k^2 + 16$

b $k^2 \geqslant 0$, therefore $k^2 + 16$ is also > 0. If $b^2 - 4ac > 0$, then $h(x)$ has two distinct real roots.

Challenge

a For distinct real roots, $b^2 - 4ac > 0$
$b^2 > 4ac$
If $a > 0$ and $c > 0$, or $a < 0$ and $c < 0$, choose b such that $b > \sqrt{4ac}$
If $a > 0$ and $c < 0$, or $a < 0$ and $c > 0$, $4ac < 0$, therefore $4ac < b^2$ for all b

b For equal roots, $b^2 - 4ac = 0$
$b^2 = 4ac$
If $4ac < 0$, then there is no value for b to satisfy $b^2 = 4ac$ as b^2 is always positive.

Quadratics 2H

1 a The bridge is 200 m above ground level, since this is the height at the centre of the bridge.

b $0.000\,12x^2 + 200 = 346$

$0.000\,12x^2 = 146$

$x^2 = \dfrac{146}{0.000\,12}$

$x = \pm\sqrt{\dfrac{146}{0.000\,12}}$

So $x = 1103$ and $x = -1103$

c length = 1103 × 2 = 2206 m

2 a $-0.01x^2 + 0.975x + 16 = 32.5$

$-0.01x^2 + 0.975x - 16.5 = 0$

Using the formula, where $a = -0.01$, $b = 0.975$ and $c = -16.5$,

$$x = \frac{-0.975 \pm \sqrt{0.975^2 - 4(-0.01)(-16.5)}}{2(-0.01)}$$

$$x = \frac{0.975 \pm \sqrt{0.290\,625}}{0.02}$$

$x = 75.7$ and $x = 21.8$ (to 3 s.f.)

21.8 mph and 75.7 mph

b $y = -0.01x^2 + 0.975x + 16$

$= -0.01(x^2 - 97.5x) + 16$

$= -0.01((x - 48.75)^2 - 2376.5625) + 16$

$= -0.01(x - 48.75)^2 + 39.765\,625$

$A = 39.77$ (to 4 s.f.), $B = 0.01$ and $C = 48.75$

c The greatest fuel efficiency is the maximum, when $x = 48.75$

48.75 mph

d When $x = 120$,

$y = -0.01(120)^2 + 0.975(120) + 16$

$= -11$

A negative fuel consumption is impossible, so this model is not valid for very high speeds.

3 a Without any fertliser, $f = 0$, so each hectare would yield 6 tonnes of grain.

3 b When $f = 20$,

$g = 6 + 0.03(20) - 0.00006(20)^2$

$= 6.576$

For an extra tonne yield, $g = 6.576 + 1$

$= 7.576$

$6 + 0.03f - 0.000\,06f^2 = 7.576$

$1.576 - 0.03f + 0.000\,06f^2 = 0$

Using the formula, where $a = 0.000\,06$, $b = -0.03$ and $c = 1.576$,

$$x = \frac{-(-0.03) \pm \sqrt{(-0.03)^2 - 4(0.000\,06)(1.576)}}{2(0.000\,06)}$$

$$x = \frac{0.03 \pm \sqrt{0.000\,521\,76}}{0.000\,12}$$

$x = 440.4$ and $x = 59.6$ (to 1 d.p.)

$59.6 - 20 = 39.6$

39.6 kilograms per hectare

4 a $t = M - 1000p$, $t = 10\,000$ when $p = £30$

$10\,000 = M - 1000 \times 30$

$M = 40\,000$

b $r = p(40\,000 - 1000p)$

$= -1000p^2 + 40\,000p$

$= -1000(p^2 - 40p)$

$= -1000((p - 20)^2 - 400)$

$= -1000(p - 20)^2 + 400\,000$

$A = 400\,000$, $B = 1000$ and $C = 20$

c $r = = -1000(p - 20)^2 + 400\,000$

maximum = £400 000 when $p = 20$

They should charge £20 per ticket.

Challenge

a d(s) = $as^2 + bs + c$
When s = 20, d = 6:
$6 = a(20)^2 + b(20) + \text{c}$
$6 = 400a + 20b + c$ (**1**)

When s = 30, d = 14:
$14 = a(30)^2 + b(30) + c$
$14 = 900a + 30b + c$ (**2**)

When s = 20, d = 24:
$24 = a(40)^2 + b(40) + c$
$24 = 1600a + 40b + c$ (**3**)

(**2**) − (**1**):
$(14 = 900a + 30b + c) - (6 = 400a + 20b + c)$
$\Rightarrow 8 = 500a + 10b$ (**4**)

(**3**) − (**1**):
$(24 = 1600a + 40b + c) - (6 = 400a + 20b + c)$
$\Rightarrow 18 = 1200a + 20b$ (**5**)

(**5**) − 2 × (**4**):
$(18 = 1200a + 20b) - 2(8 = 500a + 10b)$
$\Rightarrow 2 = 200a$, so $a = 0.01$

$8 = 500(0.01) + 10b$
$8 = 5 + 10b \Rightarrow b = 0.3$
$6 = 400(0.01) + 20(0.3) + c$
$6 = 4 + 6 + c \Rightarrow c = -4$
$a = 0.01$, $b = 0.3$ and $c = -4$

b $0.01s^2 + 0.3s - 4 = 20$
$0.01s^2 + 0.3s - 24 = 0$
Using the formula, where $a = 0.01$, $b = 0.3$ and $c = -24$,

$$x = \frac{-0.3 \pm \sqrt{0.3^2 - 4(0.01)(-24)}}{2(0.01)}$$

$$x = \frac{-0.3 \pm \sqrt{1.05}}{0.02}$$

$x = 36.2$ or -66.2 (to 3 s.f.)
The speed of the car must be positive, so is 36.2 mph.

Independent Learning A level Maths

Expectations

- You are expected to cover Chapters 1 , 2, 3 and 5 from the Edexcel AS and A level Mathematics Pure Mathematics Year 1/AS (ISBN: 9781292183398) **before** you start the A- level mathematics course and you are provided with remote learning work to enable you to do this.
- Work through the lessons according to your needs, for example you may already be confident in the topic so you can either skip straight the questions or skip the lesson altogether
- Keep evidence of this work as this can be used to demonstrate whether you are making suitable progress
- **You will be assessed on these chapters within the first term of the course and the first two chapters within the first half term**
- If you need any additional support you can contact myself (Ms Kumi) via email ekumi@chestnutgrove.org.uk

Chapter 2 – Equations and inequalities

1. **Lesson 1: Linear simultaneous equations. Watch video lesson. Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

2. **Lesson 2: Quadratic simultaneous equations. Watch video lesson . Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

3. **Lesson 3: Simultaneous equations as graphs. Watch video lesson . Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

4. **Lesson 4: Quadratic inequalities. Watch video lesson. Copy examples, complete the my turn and exercise questions. Make sure you mark all your work.**

5. **Lesson 5: Inequalities on graphs. Watch video lesson as an introduction. Work through these questions. Answers attached as file.**

For each pair of functions:

i Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the same axes.

ii Find the coordinates of any points of intersection.

iii Write down the solutions to the inequality $f(x) \leqslant g(x)$.

a $f(x) = 3x - 7$
$g(x) = 13 - 2x$

b $f(x) = 8 - 5x$
$g(x) = 14 - 3x$

c $f(x) = x^2 + 5$
$g(x) = 5 - 2x$

d $f(x) = 3 - x^2$
$g(x) = 2x - 12$

e $f(x) = x^2 - 5$
$g(x) = 7x + 13$

f $f(x) = 7 - x^2$
$g(x) = 2x - 8$

6. **Lesson 6:Regions. Watch video lesson. Watch video if needed in addition. Complete questions below. Answers on the last page . YOU CAN SKETCH BUT YOU MUST INCLUDE THE X – Y intercepts and any vertices (turning points).**

1 On a coordinate grid, shade the region that satisfies the inequalities:
$y > x - 2$, $y < 4x$ and $y \leqslant 5 - x$.

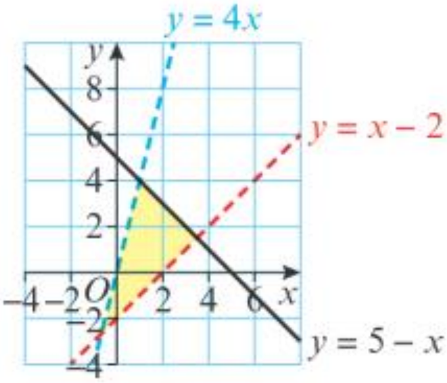
2 On a coordinate grid, shade the region that satisfies the inequalities:
$x \geqslant -1$, $y + x < 4$, $2x + y \leqslant 5$ and $y > -2$.

3 On a coordinate grid, shade the region that satisfies the inequalities:
$y < (3 - x)(2 + x)$ and $y + x \geqslant 3$.

4 On a coordinate grid, shade the region that satisfies the inequalities:
$y > x^2 - 2$ and $y \leqslant 9 - x^2$.

5 On a coordinate grid, shade the region that satisfies the inequalities:
$y > (x - 3)^2$, $y + x \geqslant 5$ and $y < x - 1$.

1



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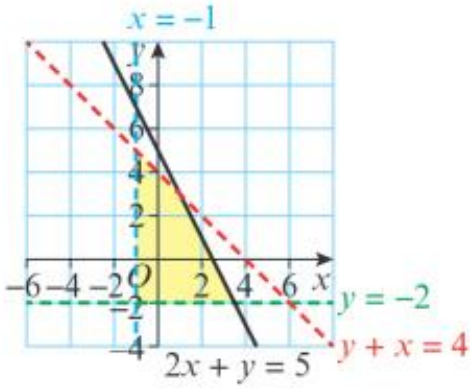

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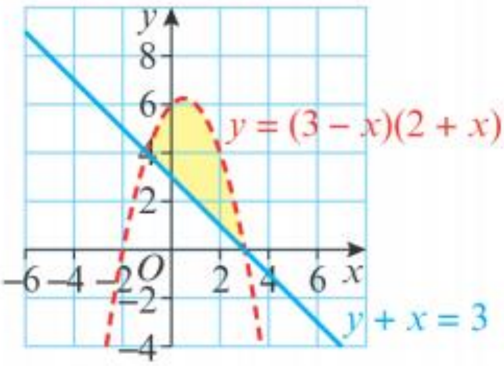

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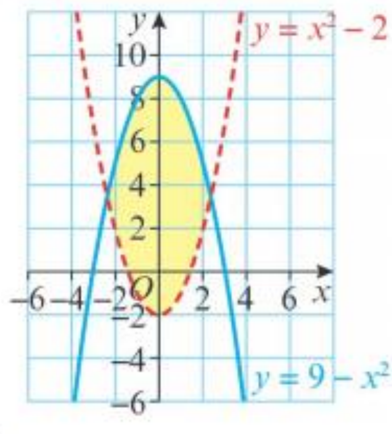
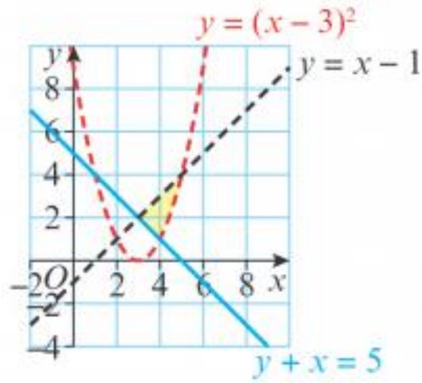


5



6 **a** For $y = x + 1$ and $y = 7 - x$:
$x + 1 = 7 - x$
$2x = 6$
$x = 3, y = 4$

For $y = 7 - x$ and $x = 1$:
$x = 1, y = 6$

For $x = 1$ and $y = x + 1$
$x = 1, y = 2$

The points of intersection are (3, 4), (1, 6) and (1, 2).

Equations and inequalities 3F

1 a $3x + 2y = 6$ **(1)**

$x - y = 5$ **(2)**

Multiply equation (**2**) by 2:

$2x - 2y = 10$ **(3)**

Add equations (**1**) and (**3**):

$5x = 16$

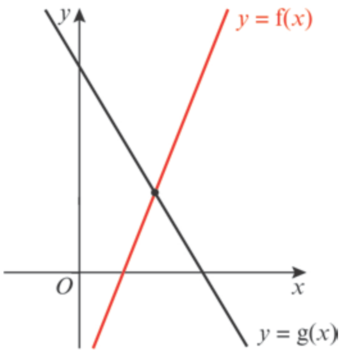
$x = \frac{16}{5}, y = -\frac{9}{5}$

The solution is $P\left(\frac{16}{5}, -\frac{9}{5}\right)$.

b $2y + 3x > x - y$ when the line L_1 is above the line L_2:

$x < \frac{16}{5}$

2 a i



ii $3x - 7 = 13 - 2x$

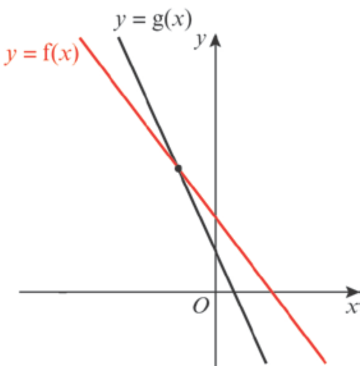
$5x = 20$

$x = 4, y = 5$

The lines intersect at (4, 5).

iii $f(x) \leq g(x)$ when the f(x) is below g(x), so $x \leq 4$

b i



2 b ii $8 - 5x = 14 - 3x$

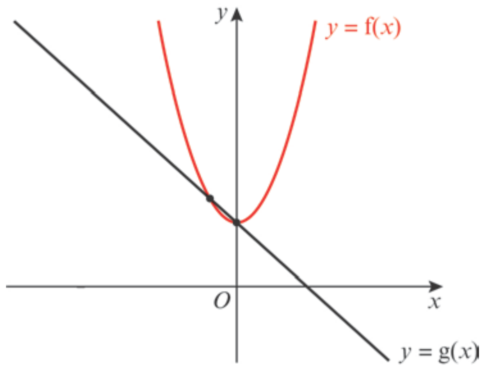
$-2x = 6$

$x = -3, y = 23$

The lines intersect at (−3, 23).

iii $f(x) \leq g(x)$ when f(x) is below g(x), so $x \geq -3$

c i



ii $x^2 + 5 = 5 - 2x$

$x^2 + 2x = 0$

$x(x + 2) = 0$

$x = 0$ or $x = -2$

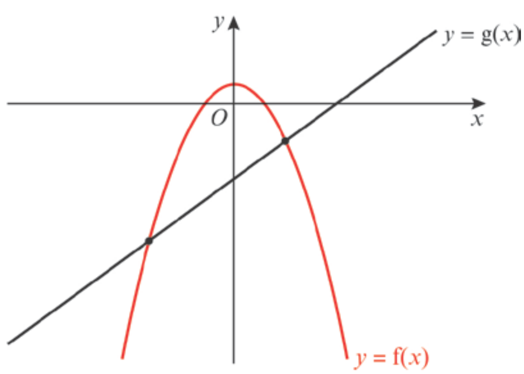
When $x = 0, y = 5$

When $x = -2, y = 9$

The lines intersect at (0, 5) and (−2, 9).

iii $f(x) \leq g(x)$ when f(x) is below g(x), so $-2 \leq x \leq 0$

d i



2 d ii $3 - x^2 = 2x - 12$

$x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$

$x = -5$ or $x = 3$

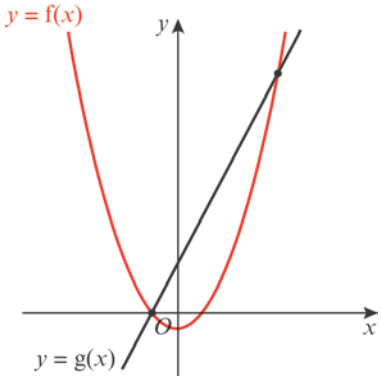
When $x = -5$, $y = -22$

When $x = 3$, $y = -6$

The lines intersect at $(-5, -22)$ and $(3, -6)$.

iii $f(x) \leq g(x)$ when $f(x)$ is below $g(x)$, so $x \leq -5$ or $x \geq 3$

e i



ii $x^2 - 5 = 7x + 13$

$x^2 - 7x - 18 = 0$

$(x - 9)(x + 2) = 0$

$x = 9$ or $x = -2$

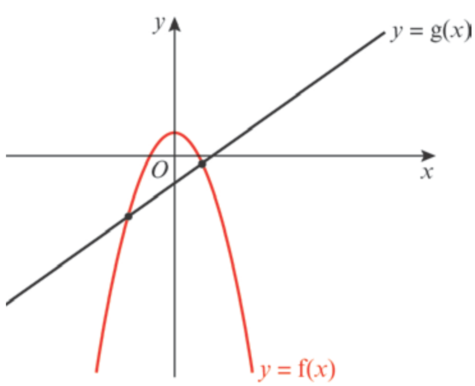
When $x = 9$, $y = 76$

When $x = -2$, $y = -1$

The lines intersect at $(-2, -1)$ and $(9, 76)$.

iii $f(x) \leq g(x)$ when $f(x)$ is below $g(x)$, so $-2 \leq x \leq 9$

f i



f ii $7 - x^2 = 2x - 8$

$x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$

$x = -5$ or $x = 3$

When $x = -5$, $y = -18$

When $x = 3$, $y = -2$

The lines intersect at $(-5, -18)$ and $(3, -2)$

iii $f(x) \leq g(x)$ when $f(x)$ is below $g(x)$, so $x \leq -5$ or $x \geq 3$

3 a $3x^2 - 2x - 1 = x + 5$

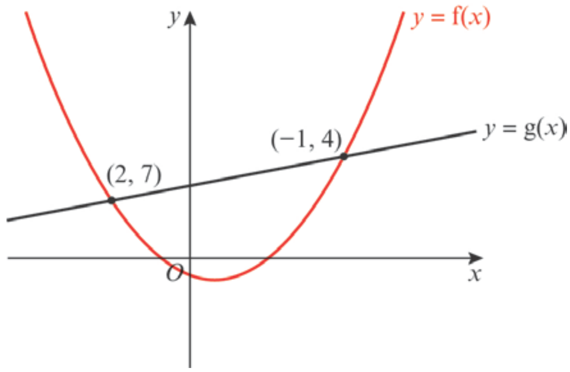
$3x^2 - 3x - 6 = 0$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2, x = -1$

The points of intersection are $(2, 7)$ and $(-1, 4)$.



So the required values are $-1 < x < 2$

b $2x^2 - 4x + 1 = 3x - 2$

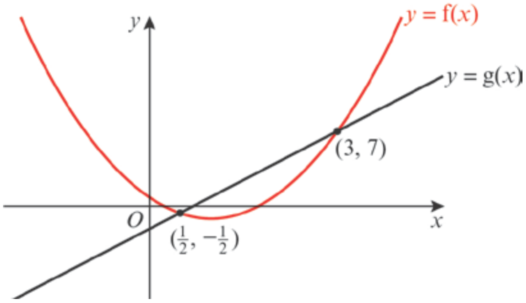
$2x^2 - 7x + 3 = 0$

$(2x - 1)(x - 3) = 0$

$x = \frac{1}{2}$ or $x = 3$

The points of intersection are $(\frac{1}{2}, -\frac{1}{2})$ and $(3, 7)$.

3 b



So the required values are $\frac{1}{2} < x < 3$

c $5x - 2x^2 - 4 = -2x - 1$

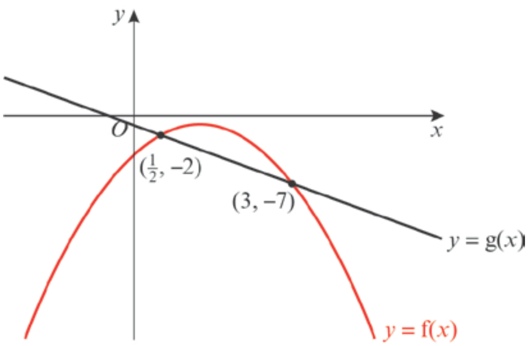
$2x^2 - 7x + 3 = 0$

$(2x - 1)(x - 3) = 0$

$x = \frac{1}{2}$ or $x = 3$

The points of intersection are

$(\frac{1}{2}, -2)$ and $(3, -7)$.



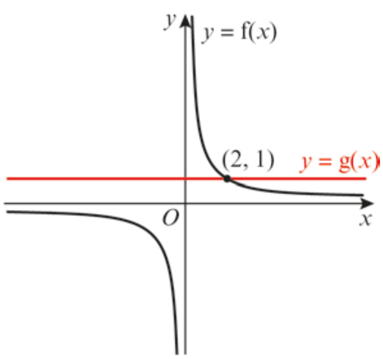
So the required values are

$x < \frac{1}{2}$ or $x > 3$

d $\dfrac{2}{x} = 1$

$x = 2$

Point of intersection is (2, 1)



d So the required values are

$x < 0$ or $x > 2$

e $\dfrac{3}{x^2} - \dfrac{4}{x} = -1$

Multiply both sides by x^2

$3 - 4x = -x^2$

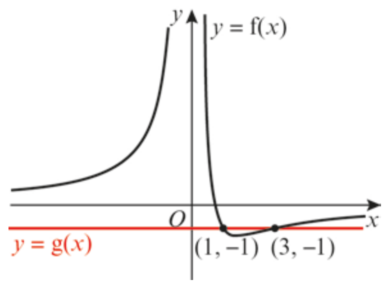
$x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0$

$x = 1$ or $x = 3$

Points of intersection are

$(1, -1)$ and $(3, -1)$



So the required values are $1 < x < 3$

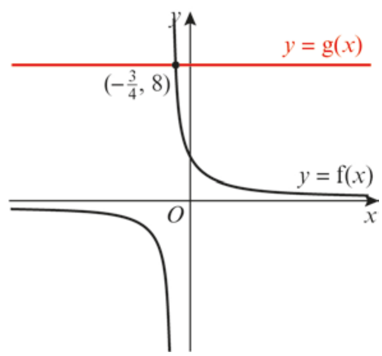
f $\dfrac{2}{x+1} = 8$

$2 = 8(x + 1)$

$8x + 6 = 0$

$x = -\frac{3}{4}$

Point of intersection is $(-\frac{3}{4}, 8)$



So the required values are

$x < -1$ or $x > -\frac{3}{4}$

Challenge

a $x^2 - 4x - 12 = 6 + 5x - x^2$
$2x^2 - 9x - 18 = 0$
$(2x + 3)(x - 6) = 0$
$x = -\frac{3}{2}$ or $x = 6$
The points of intersection are
$(-\frac{3}{2}, -\frac{15}{4})$ and (6, 0).

b So the required values are $-\frac{3}{2} < x < 6$
$\{x: -\frac{3}{2} < x < 6\}$

Independent Learning A level Maths

Expectations

- You are expected to cover Chapters 1 , 2, 3 and 5 from the Edexcel AS and A level Mathematics Pure Mathematics Year 1/AS (ISBN: 9781292183398) **before** you start the A- level mathematics course and you are provided with remote learning work to enable you to do this.
- Work through the lessons according to your needs, for example you may already be confident in the topic so you can either skip straight the questions or skip the lesson altogether
- Keep evidence of this work as this can be used to demonstrate whether you are making suitable progress
- **You will be assessed on these chapters within the first term of the course and the first two chapters within the first half term**
- If you need any additional support you can contact myself (Ms Kumi) via email ekumi@chestnutgrove.org.uk

Chapter 5 – Straight lines

1. **Lesson 1: y=mx +c (Gradient of a line given two points). Watch gradient of a line from a graph lesson and Gradient of a line given two points. Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

2. **Lesson 2: Equation of a line. Watch equation of a line lesson and equation of a line 2. Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

3. **Lesson 3: Parallel and perpendicular lines. Watch Parallel lines lesson and Perpendicular lines. Copy all the examples, complete all the my turn and exercise questions. Make sure you mark all your work.**

4. **Lesson 4: Length and area. Watch Length and area lesson. Complete questions below. Answers attached as file.**

Exercise 5G

1 Find the distance between these pairs of points:

a (0, 1), (6, 9) **b** (4, −6), (9, 6) **c** (3, 1), (−1, 4)

2 Consider the points $A(-3, 5)$, $B(-2, -2)$ and $C(3, -7)$. Determine whether the line joining the points A and B is congruent to the line joining the points B and C.

Hint Two line segments are congruent if they are the same length.

E/P **7** A point P lies on the line with equation $y = 4 - 3x$. The point P is a distance $\sqrt{34}$ from the origin. Find the two possible positions of point P. **(5 marks)**

P **8** The vertices of a triangle are $A(2, 7)$, $B(5, -6)$ and $C(8, -6)$.

a Show that the triangle is a scalene triangle.

b Find the area of the triangle ABC.

Notation Scalene triangles have three sides of different lengths.

Problem-solving Draw a sketch and label the points A, B and C. Find the length of the base and the height of the triangle.

E **11** The points $R(5, -2)$ and $S(9, 0)$ lie on the straight line l_1 as shown.

a Work out an equation for straight line l_1. **(2 marks)**

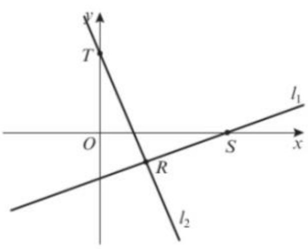
The straight line l_2 is perpendicular to l_1 and passes through the point R.

b Work out an equation for straight line l_2. **(2 marks)**

c Write down the coordinates of T. **(1 mark)**

d Work out the lengths of RS and TR leaving your answer in the form $k\sqrt{5}$. **(2 marks)**

e Work out the area of $\triangle RST$. **(2 marks)**



5. Lesson 5: Modelling with straight lines. Rewatch Length and area lesson if needed. Complete questions below. Answers attached as file.

(E/P) **5** A website designer charges a flat fee and then a daily rate in order to design new websites for companies.

Company A's new website takes 6 days and they are charged £7100.

Company B's new website take 13 days and they are charged £9550.

Hint Let $(d_1, C_1) = (6, 7100)$ and $(d_2, C_2) = (13, 9550)$.

a Write an equation linking days, d and website cost, C in the form $C = ad + b$. **(3 marks)**

b Interpret the values of a and b. **(2 marks)**

c The web designer charges a third company £13 400. Calculate the number of days the designer spent working on the website. **(1 mark)**

(E/P) **6** The average August temperature in Exeter is 20 °C or 68 °F. The average January temperature in the same place is 9 °C or 48.2 °F.

a Write an equation linking Fahrenheit F and Celsius C in the form $F = aC + b$. **(3 marks)**

b Interpret the values of a and b. **(2 marks)**

c The highest temperature recorded in the UK was 101.3 °F. Calculate this temperature in Celsius. **(1 mark)**

d For what value is the temperature in Fahrenheit the same as the temperature in Celsius? **(3 marks)**

(P) **7** In 2004, in a city, there were 17 500 homes with internet connections. A service provider predicts that each year an additional 750 homes will get internet connections.

a Write a linear model for the number of homes n with internet connections t years after 2004.

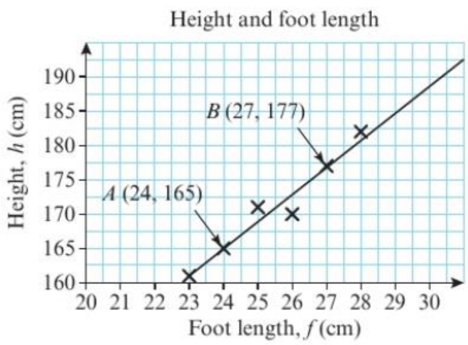
b Write down one assumption made by this model.

(E/P) **8** The scatter graph shows the height h and foot length f of 8 students. A line of best fit is drawn on the scatter graph.

a Explain why the data can be approximated to a linear model. **(1 mark)**

b Use points A and B on the scatter graph to write a linear equation in the form $h = af + b$. **(3 mark)**

c Calculate the expected height of a person with a foot length of 26.5 cm. **(1 mark)**



Straight line graphs 5G

1 a $(x_1, y_1) = (0, 1), (x_2, y_2) = (6, 9)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(6-0)^2 + (9-1)^2}$
$= \sqrt{6^2 + 8^2}$
$= \sqrt{36 + 64}$
$= \sqrt{100}$
$= 10$

b $(x_1, y_1) = (4, -6), (x_2, y_2) = (9, 6)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(9-4)^2 + (6-(-6))^2}$
$= \sqrt{5^2 + 12^2}$
$= \sqrt{25 + 144}$
$= \sqrt{169}$
$= 13$

c $(x_1, y_1) = (3, 1), (x_2, y_2) = (-1, 4)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(-1-3)^2 + (4-1)^2}$
$= \sqrt{(-4)^2 + 3^2}$
$= \sqrt{16 + 9}$
$= \sqrt{25}$
$= 5$

d $(x_1, y_1) = (3, 5), (x_2, y_2) = (4, 7)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(4-3)^2 + (7-5)^2}$
$= \sqrt{1^2 + 2^2}$
$= \sqrt{1 + 4}$
$= \sqrt{5}$

e $(x_1, y_1) = (0, -4), (x_2, y_2) = (5, 5)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(5-0)^2 + (5-(-4))^2}$
$= \sqrt{5^2 + 9^2}$
$= \sqrt{25 + 81}$
$= \sqrt{106}$

f $(x_1, y_1) = (-2, -7), (x_2, y_2) = (5, 1)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(5-(-2))^2 + (1-(-7))^2}$
$= \sqrt{(5+2)^2 + (1+7)^2}$
$= \sqrt{7^2 + 8^2}$
$= \sqrt{49 + 64}$
$= \sqrt{113}$

2 $A(-3, 5)$, $B(-2, -2)$ and $C(3, -7)$.
Lines are congruent if they are the same length.

Using the distance formula, and taking the unknown length as d:
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For the line AB:
$(x_1, y_1) = (-3, 5), (x_2, y_2) = (-2, -2)$
$AB = \sqrt{((-2)-(-3))^2 + ((-2)-5)^2}$
$= \sqrt{1^2 + (-7)^2}$
$= \sqrt{50}$

For the line BC:
$(x_1, y_1) = (-2, -2), (x_2, y_2) = (3, -7)$
$BC = \sqrt{(3-(-2))^2 + ((-7)-(-2))^2}$
$= \sqrt{5^2 + (-5)^2}$

2 $BC = \sqrt{50}$
$AB = BC$, $\therefore$ they are congruent.

3 $P(11, -8)$, $Q(4, -3)$ and $R(7, 5)$.

Using the distance formula, and taking the unknown length as d:
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For the line PQ:
$(x_1, y_1) = (11, -8), (x_2, y_2) = (4, -3)$
$PQ = \sqrt{(4-11)^2 + ((-3)-(-8))^2}$
$= \sqrt{(-7)^2 + 5^2}$
$= \sqrt{74}$

For the line QR:
$(x_1, y_1) = (4, -3), (x_2, y_2) = (7, 5)$
$QR = \sqrt{(7-4)^2 + (5-(-3))^2}$
$= \sqrt{3^2 + 8^2}$
$= \sqrt{73}$

$PQ \neq QR$, therefore the two lines are not congruent.

4 The distance between the points $(-1, 13)$ and $(x, 9)$ is $\sqrt{65}$.

Using the distance formula, and taking the length as d:
$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$65 = (x-(-1))^2 + (9-13)^2$
$65 = (x+1)^2 + (-4)^2$
$65 = x^2 + 2x + 1 + 16$
$x^2 + 2x - 48 = 0$
$(x + 8)(x - 6) = 0$
$x = -8$ or $x = 6$

5 The distance between the points $(2, y)$ and $(5, 7)$ is $3\sqrt{10}$.

Using the distance formula, and taking the length as d:
$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$90 = (5-2)^2 + (7-y)^2$
$90 = 3^2 + 49 - 14y + y^2$
$y^2 - 14y - 32 = 0$
$(y - 16)(y + 2) = 0$
$y = 16$ or $y = -2$

6 a l_1: $y = 2x + 4$, gradient = 2
l_2: $6x - 3y - 9 = 0$

Rearrange line l_2 to give:
$3y = 6x - 9$
$y = 2x - 3$, gradient = 2

Lines l_1 and l_2 both have gradient 2 so they are parallel.

b Line l_3 is perpendicular to line l_1 so has gradient $-\frac{1}{2}$.

It also passes through the point (3, 10).

$y - y_1 = m(x - x_1)$
$y - 10 = -\frac{1}{2}(x - 3)$
$2y - 20 = -x + 3$
$x + 2y - 23 = 0$ is the equation of l_3

c l_2: $y = 2x - 3$
l_3: $2y = -x + 23$

Dividing through by 2:
$y = -\frac{1}{2}x + \frac{23}{2}$

At the point of intersection the two expressions for y are equal, so:
$2x - 3 = -\frac{1}{2}x + \frac{23}{2}$

6 c Then multiplying through by 2:

$4x - 6 = -x + 23$

$5x = 29$

$x = \frac{29}{5}$

Substituting $x = \frac{29}{5}$ into $y = 2x - 3$:

$y = 2\left(\frac{29}{5}\right) - 3$

$= \frac{43}{5}$

The point of intersection of the lines l_1 and l_2 is $\left(\frac{29}{5}, \frac{43}{5}\right)$.

d l_1 and l_2 are parallel so the shortest distance between them is the perpendicular distance.

l_3 is perpendicular to l_1 and therefore is perpendicular to l_2.

l_2 and l_3 intersect at (, $\frac{43}{5}$).

Now work out the point of intersection for lines l_1 and l_3.

l_1: $y = 2x + 4$

l_3: $y = -\frac{1}{2}x + \frac{23}{2}$

$2x + 4 = -\frac{1}{2}x + \frac{23}{2}$

$4x + 8 = -x + 23$

$5x = 15, x = 3$

When $x = 3$, $y = 10$

The point of intersection of the lines l_1 and l_3 is (3, 10).

Now find the distance, d, between (3, 10) and $\left(\frac{29}{5}, \frac{43}{5}\right)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left(\tfrac{29}{5} - 3\right)^2 + \left(\tfrac{43}{5} - 10\right)^2}$$

$$= \sqrt{\left(\tfrac{14}{5}\right)^2 + \left(-\tfrac{7}{5}\right)^2}$$

$$= \sqrt{\tfrac{245}{25}}$$

$$= \tfrac{1}{5}\sqrt{245}$$

$$= \tfrac{1}{5}\sqrt{49 \times 5}$$

6 d $d = \dfrac{7\sqrt{5}}{5}$

The perpendicular distance between l_1 and l_2 is $\frac{7}{5}\sqrt{5}$.

7 Point P is on the line $y = -3x + 4$.

Its distance, d, from (0, 0) is $\sqrt{34}$.

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$34 = (x - 0)^2 + (y - 0)^2$$

$$34 = x^2 + y^2$$

Solve $34 = x^2 + y^2$ and $y = -3x + 4$ simultaneously.

$34 = x^2 + (-3x + 4)^2$

$34 = x^2 + 9x^2 - 24x + 16$

$10x^2 - 24x - 18 = 0$

$5x^2 - 12x - 9 = 0$

$(5x + 3)(x - 3) = 0$

$x = -\frac{3}{5}$ or $x = 3$

When $x = \frac{3}{5}$, $y = -3\left(-\frac{3}{5}\right) + 4$

$y = \frac{29}{5}$

When $x = 3$, $y = -3(3) + 4$

$y = -5$

So P is the point $\left(-\frac{3}{5}, \frac{29}{5}\right)$ or (3, −5).

8 a In a scalene triangle, all three sides have different lengths: $AB \neq BC \neq AC$.

A(2, $\frac{29}{5}$ 7), B(5, −6) and C(8, −6).

$$AB = \sqrt{(5-2)^2 + ((-6)-7)^2}$$

$$= \sqrt{3^2 + (-13)^2}$$

$$= \sqrt{178}$$

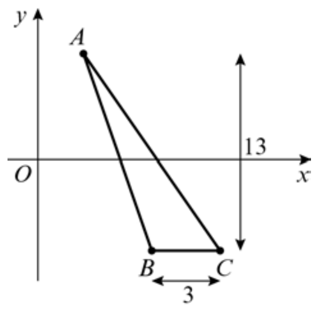
$$BC = \sqrt{(8-5)^2 + ((-6)-(-6))^2}$$

$$= \sqrt{3^2 + 0^2}$$

$$= 3$$

8a $AC = \sqrt{(8-2)^2 + ((-6)-7)^2}$
$= \sqrt{6^2 + (-13)^2}$
$= \sqrt{205}$
$AB \neq BC \neq AC$ therefore ABC is a scalene triangle.

b Draw a sketch and labels the points A, B and C.



Area of ΔABC $= \frac{1}{2}bh$
$= \frac{1}{2} \times 3 \times 13$
$= 19.5$

9 a l_1: $y = 7x - 3$
l_2: $4x + 3y - 41 = 0$

Substituting l_1 into l_2 gives:
$4x + 3(7x - 3) - 41 = 0$
$4x + 21x - 9 - 41 = 0$
$25x = 50$
$x = 2$

Substituting $x = 2$ into $y = 7x - 3$ gives $y = 11$.
A is the point (2, 11).

b When l_2 crosses the x-axis, $y = 0$.
So $4x + 3(0) - 41 = 0$
$4x = 41$
$x = \frac{41}{4}$
B is the point $(\frac{41}{4}, 0)$.

9 c The base of ΔAOB is $\frac{41}{4}$
The height of ΔAOB is 11

Area $\Delta AOB = \frac{1}{2} \times \frac{41}{4} \times 11$
$= \frac{451}{8}$

10 a l_1: $4x - 5y - 10 = 0$ intersects the x-axis at A, so $y = 0$.

$4x - 5(0) - 10 = 0$
$4x = 10$
$x = \frac{5}{2}$
A is the point $\left(\frac{5}{2}, 0\right)$.

b l_2: $4x - 2y + 20 = 0$ intersects the x-axis at B, so $y = 0$.

$4x - 2(0) + 20 = 0$
$4x = -20$
$x = -5$

B is the point $(-5, 0)$.

c l_1: $4x = 5y + 10$, l_2: $4x = 2y - 20$

Where the lines intersect:
$5y + 10 = 2y - 20$
$3y = -30$
$y = -10$

Substituting $y = -10$ into $4x = 5y + 10$:
$4x = -40$
$x = -10$

l_1 and l_2 intersect at the point $(-10, -10)$.

d The base of ΔABC is $7\frac{1}{2}$.
The height of the triangle is 10.
Area ΔABC $= \frac{1}{2} \times 7\frac{1}{2} \times 10$
$= \frac{75}{2}$

11 a $R(5, -2)$ and $S(9, 0)$ lie on a straight line.

The gradient, m, of the line is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-2)}{9-5} = \frac{2}{4} = \frac{1}{2}$$

The equation of the line is:

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = \tfrac{1}{2}(x-5)$$
$$y + 2 = \tfrac{1}{2}x - \tfrac{5}{2}$$
$$y = \tfrac{1}{2}x - \tfrac{9}{2}$$

b l_2 is perpendicular to l_1 so has gradient -2.

The equation of the line is:

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -2(x-5)$$
$$y + 2 = -2x + 10$$
$$y = -2x + 8$$

c The y-intercept for line l_2 is 8.
T is the point (0, 8).

d
$$RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(9-5)^2 + (0-(-2))^2} = \sqrt{4^2 + 2^2} = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$$

11 d
$$TR = \sqrt{(5-0)^2 + ((-2)-8)^2} = \sqrt{5^2 + (-10)^2} = \sqrt{125} = 5\sqrt{5}$$

e The base of ΔRST is RS, $2\sqrt{5}$.
The height of ΔRST is RT, $5\sqrt{5}$.

Area $\Delta RST = \tfrac{1}{2} \times 2\sqrt{5} \times 5\sqrt{5}$
$= 25$

12 a l_1 has gradient $m = -\tfrac{1}{4}$ and passes through the point $(-4, 14)$.

$$y - y_1 = m(x - x_1)$$
$$y - 14 = -\tfrac{1}{4}(x-(-4))$$
$$4y - 56 = -x - 4$$
$$x + 4y - 52 = 0$$

b When l_1 crosses the y-axis, $x = 0$.
$$0 + 4y - 52 = 0$$
$$y = 13$$
A is the point (0, 13).

c l_2 has gradient, $m = 3$ and passes through the point (0, 0).
$y = 3x$

To find point B, substitute l_2 into l_1:
$$x + 4(3x) - 52 = 0$$
$$13x - 52 = 0$$
$$x = 4$$

Substitute $x = 4$ into $y = 3x$.
$y = 12$
B is the point (4, 12).

d The base of ΔOAB is $OA = 13$.
The height of ΔOAB is the distance of B from the y-axis $= 4$
Area $\Delta OAB = \tfrac{1}{2} \times 13 \times 4 = 26$

Straight line graphs 5H

1 a i Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{600 - 200}{12 - 4}$$
$$= \frac{400}{8}$$
$$= 50$$
$$k = 50$$

Direct proportion equations go through the origin so $c = 0$.

ii $d = kt + c \quad k = 50, c = 0$

$d = 50t$

b i Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{9 - 3}{30 - 10}$$
$$= \frac{6}{20}$$
$$= \frac{3}{10}$$

$k = \frac{3}{10}$

ii $C = kt + c \quad k = \frac{3}{10}, c = 0$

$C = \frac{3}{10}t$

c i Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$

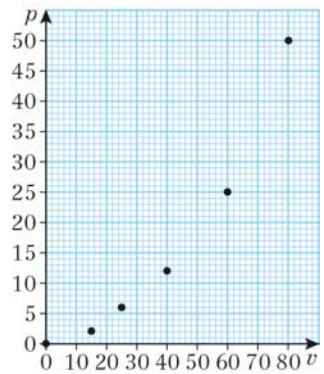
$$= \frac{18 - 6}{30 - 10}$$
$$= \frac{12}{20}$$
$$= \frac{3}{5}$$

$k = \frac{3}{5}$

ii $p = kt + c \quad k = \frac{3}{5}, c = 0$

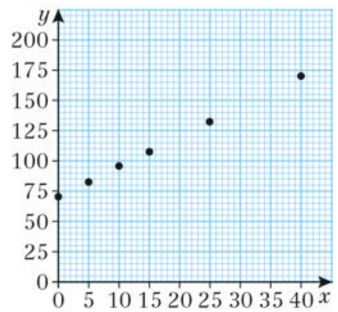
$p = \frac{3}{5}t$

2 a



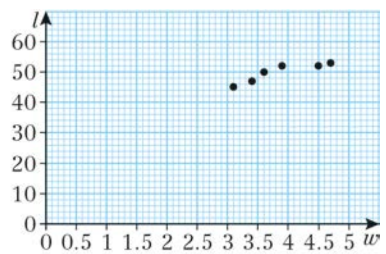
The points do not lie on a straight line, so a linear model is not appropriate.

b



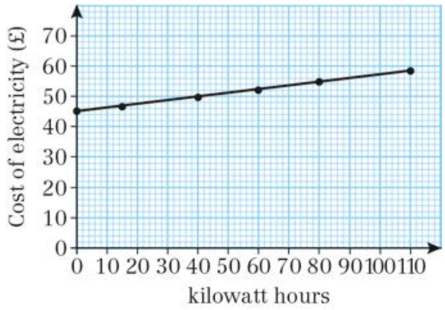
The points lie on a straight line so a linear model is appropriate.

c



The points do not lie on a straight line, so a linear model is not appropriate.

3 a



b The points lie on a straight line so a linear model is appropriate.

c Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{58.2 - 45}{110 - 0}$

$= \dfrac{13.2}{110}$

$= 0.12$

$E = ah + b$

a is the gradient = 0.12

b is the y-intercept = 45

$E = 0.12h + 45$

d a = £0.12, this is the cost of 1 kilowatt hour of electricity.

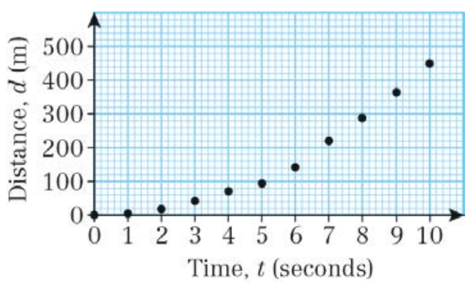
b = £45, this is the fixed charge for the electricity supply (per month or per quarter).

e When $h = 65$:

$E = 0.12(65) + 45$

= £52.80

4 a



b The points do not lie on a straight line, so a linear model is not appropriate.

5 a $(x_1, y_1) = (6, 7100)$

$(x_2, y_2) = (13, 9550)$

Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{9550 - 7100}{13 - 6}$

$= \dfrac{2450}{7}$

$= 350$

$C = ad + b$

$C = 350d + b$

Substituting $d = 6$ and $C = 7100$ into $C = 350d + b$ gives:

$7100 = 350(6) + b$

$b = 5000$

$C = 350d + 5000$

b a = £350, this is the daily fee charged by the web designer.

b = £5000, this is the flat rate fee charged by the web designer.

c Substitute $C = 13\,400$ into $C = 350d + 5000$ to give:

$13\,400 = 350d + 5000$

$d = 24$

The designer spent 24 days working on the website.

6 a $(x_1, y_1) = (9, 48.2)$, $(x_2, y_2) = (20, 68)$

Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{68 - 48.2}{20 - 9}$

$= \dfrac{19.8}{11}$

$= 1.8$

$F = 1.8C + b$

Substituting $C = 9$ and $F = 48.2$ into $F = 1.8C + b$ gives:

$48.2 = 1.8(9) + b$

$b = 32$

$F = 1.8C + 32$

6 b $a = 1.8$ which is the increase in the Fahrenheit temperature for every 1 degree increase in the Celsius temperature.
$b = 32$ which is the Fahrenheit temperature when the Celsius temperature is zero.

c Substitute $F = 101.3$ into $F = 1.8C + 32$ to give:
$101.3 = 1.8C + 32$
$C = 38.5$
The temperature 101.3 °F is 38.5 °C.

d $F = 1.8C + 32$
When $F = C$:
$F = 1.8F + 32$
$-0.8F = 32$
$F = -40$
−40 °F is the same as −40 °C.

7 a Gradient = 750
Intercept on the vertical axis = 17 500
$n = 750t + 17\,500$

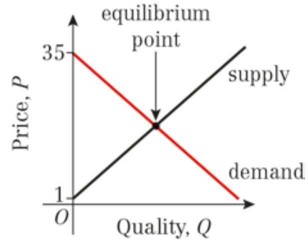
b The assumption is that the number of homes receiving internet connection will increase by the same amount each year.

8 a The data can be approximated to a linear model as all of the points lie close to the line of best fit shown.

b Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$
$= \dfrac{177 - 165}{27 - 24}$
$= 4$
$h = 4f + b$
Substituting $f = 24$ and $h = 165$ into $h = 4f + b$ gives:
$165 = 4(24) + b$
$b = 69$
$h = 4f + 69$

8 c Substituting $f = 26.5$ into $h = 4f + 69$ gives:
$h = 4(26.5) + 69$
$= 175$ cm

9 a



b Solve $P = -\frac{3}{4}Q + 35$ and $P = \frac{2}{3}Q + 1$ simultaneously:
$-\frac{3}{4}Q + 35 = \frac{2}{3}Q + 1$
$34 = \frac{17}{12}Q$
$Q = 24$
Substituting $Q = 24$ into P gives:
$P = \frac{2}{3}Q + 1$: P
$= \frac{2}{3}(24) + 1$
$= 17$
$P = 17$, $Q = 24$